Lec 8: 02/11/2010

Standard Cosmological Model:

Enormous data from various experiments has led to the emergence of a "Stadard Cosmological Model" over the past decade. This model describes the universe and its evolution from very early moments to the present time based on a few parameters.

Lets consider the Friedmann equations:

$$H^2 = \frac{8\pi}{3} G \sum_i \rho_i - \frac{k}{a^2}$$

$$qH^{-2} = -\frac{4\pi}{3} G \sum_i (\rho_i + 3P_i)$$

Here H and q are the Hubble expansion rate and the deceleration parameter respectively. The sum over "i" takes all components in the energy density of the universe into account.

We know of three components based on the observations, matter (ρ_m), radiation (ρ_{rad}) and dark energy (ρ_{DE}). The evidence for dark energy has been around for 12 years now. The term generally stands for a component that dominates the universe today and drives an accelerated expansion, which requires that $p_{DE} < -\frac{1}{3}\rho_{DE}$ (or $\omega_{DE} < -\frac{1}{3}$).

As we discussed in the previous lectures, observations of nearby objects determine H_0 and q_0 (subscript "0" denotes the value of corresponding quantity at the present epoch). We would also like to know $\rho_{m,0}$, $\rho_{rad,0}$, $\rho_{DE,0}$ (also ω_{DE}) and the geometry of the universe (encoded in "k"). The two Friedmann equations are not sufficient for determination of these parameters from H_0 and q_0. We therefore need other observations to pin down these parameters.

A useful way to express the energy budget of the universe is to normalize the energy density by $\frac{3H^2}{8\pi G}$:

$$\Omega = \frac{\rho_{tot}}{\frac{3H^2}{8\pi G}\ (\equiv \rho_{crit})}$$

$$\Omega = \Omega_m + \Omega_{rad} + \Omega_{DE}$$

$$\Omega_m = \frac{\rho_m}{\frac{3H^2}{8\pi G}}, \quad \Omega_{rad} = \frac{\rho_{rad}}{\frac{3H^2}{8\pi G}}, \quad \Omega_{DE} = \frac{\rho_{DE}}{\frac{3H^2}{8\pi G}}$$

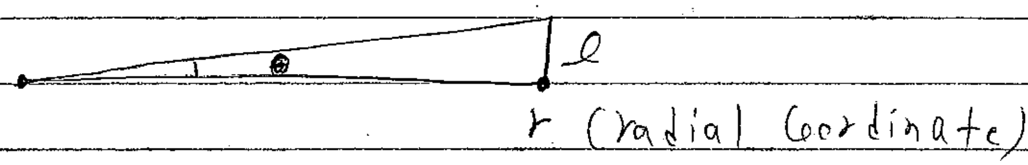
We note that for a flat universe ($k=0$) we have $H^2 = \frac{8\pi}{3} G \rho_{tot}$, and hence $\Omega = 1$. In fact, $\Omega = 1$ at all times for a flat universe.

Therefore $\Omega_m + \Omega_{rad} + \Omega_{DE} = 1$ at all times in a flat universe, but individually they vary in time simply because $\rho_m, \rho_{rad}, \rho_{DE}$ have different ~~depen~~ dence of the scale factor "a".

What we would like to know is Ω_0 ($\Omega_0 = 1$ for $k=0$,

$\Omega > 1$ for $k=+1$, $\Omega < 1$ for $k=-1$) and $\Omega_{m,0}$, $\Omega_{rad,0}$, $\Omega_{DE,0}$. Lets discuss what we know about these quantities and from which observation we infer these values.

1- Geometry of the universe (Ω_0). In order to prob. the geometry we need to look at very far objects ($z \gg 1$). Note that an open or a closed universe look like flat locally. The best evidence for a flat universe comes from CMB. The location of the first acoustic peak in the power spectrum of CMB from the Boomerang and Maxima experiments in 2000 indicated that $k=0$. How the location can be related to the geometry will be discussed in more detail later. The point is if we know the size of an objects and its angular distance, then we can find the geometry of the space. In a static universe we have:



$\theta = \frac{r}{l}$ $k=0$, $\theta = \frac{\sin r}{l}$ $k=+1$, $\theta = \frac{\sinh r}{l}$ $k=-1$

Once we know l, r, θ then we can see which of the three relationships are satisfied.

So far $\Omega_0 = 1$ (flat universe) is consistent with the data (also from more recent experiments)

2- Matter content of the universe ($\Omega_{m,0}$). As we will see later, there are various lines of evidence that beside the luminos baryonic matter there also exists a dark non baryonic component called "dark matter". Note that luminos and dark components are similar in that the energy density of both redshifts $\propto a^{-3}$. But they are different in that the dark matter decouples from photons much earlier than the baryonic matter (we will discuss this in more detail later on).

The early decoupling of dark matter has important implications for the shape of CMB power spectrum and structure formation. It implies that the inhomogeneities in the dark matter start to grow earlier and form gravitational wells. Baryonic matter then falls into these wells when it decouples from photons (at the "recombination epoch", which happened when the universe was ~400,000 years old).

$\Omega_{m,0}$ can be inferred from joint CMB and SNe data. Note that accelerated expansion has begun recently (driven by ρ_{DE}), while CMB photons arrive from very old times. This implies that CMB photons experience both matter domination and dark energy domination epochs. Therefore CMB data alone cannot pinpoint Ω_m (or Ω_{DE}). We know that $\Omega_{m,0} + \Omega_{DE,0} \simeq 1$ (flat universe as we just discussed $\Omega_{rad,0}$ is negligible as we will see). On the other hand, SNe data are sensitive to $\Omega_{DE} - \Omega_m$. In consequence, joint CMB and SNe data can pinpoint $\Omega_{m,0}$ and $\Omega_{DE,0}$ separately:

$$\underline{\underline{\Omega_{DE,0} \simeq 0.73}}, \quad \underline{\underline{\Omega_{m,0} \simeq 0.27}}$$

Also:

$$\Omega_{m,0} = \Omega_{b,0} + \Omega_{DM,0}$$

From CMB and LSS (large scale structure) data we can infer:

(half in "dark baryons", more discussion later on)

$\underline{\underline{\Omega_{b,0} \simeq 0.04}}$, $\underline{\underline{\Omega_{DM,0} \simeq 0.23}}$

In addition, dark matter is "cold", which means that dark matter particles were non-relativisti when they decoupled from the primordial plasma.

3_ Equation of State of dark energy (ω_{DE}). As we saw, all needed for an accelerated expansion is to have a component that has dominated the universe recently and has $\omega < \frac{1}{3}$. An important question is the value of ω_{DE}. A pure cosmological constant has $\omega = -1$. So far all observation (CMB and SNe included) are consistent with a cosmological constant $\underline{\omega_{DE} = -1}$. There are various proposals for missions to furthrer probe the equation of state of dark energy and whether ω is time-dependent (the so-called joint dark energy missions, JDEM).

4- Contribution from radiation ($\Omega_{rad,0}$). This can be easily calculated. At the present time, the only relativistic particles in the universe are photons. The energy density in the CMB photons is found to be:

$$\rho_{rad,0} = 2 \times \frac{\pi^2}{30} T^4 \qquad T \simeq 2.72^\circ k$$

$$\boxed{\rho_{rad,0} = 8.1 \times 10^{-31} \frac{kg}{m^3} \simeq 5 \times 10^{-4} \frac{GeV}{m^3}}$$

The total energy density at the present time is:

$$\rho_{tot,0} \simeq 10 \frac{GeV}{m^3}$$

It is easy to see that $\rho_{rad,0}$ is totally negligible. However, it becomes more and more important as we go back in time.